Non-linear approximation with respect to the Faber-Schauder dictionary

We apply the theory of unconditional Faber-Schauder characterizations of non-periodic function spaces provided in [1] and extend it to Sobolev spaces $S^r_pW([0,1]^d)$ with dominating mixed smoothness. These characterizations yield intrinsic discretizations of $S^r_pW([0,1]^d)$ and Besov spaces $S^r_{p,\theta}B([0,1]^d)$ that allow us to study best m-term approximation with respect to the Faber-Schauder dictionary where the error is measured in $L_q([0,1]^d)$. Compared to linear (sampling) approximation in case $p < q$ the exact values of p and q do not affect the asymptotic rate. We obtain always

$$\sigma_m(S^r_pW([0,1]^d), \mathbb{F})_{L_q([0,1]^d)} \lesssim m^{-r}(\log m)^{(d-1)(r+\frac{1}{2})},$$

especially in the case $q = \infty$. For mixed Besov spaces $S^r_{p,\theta}B([0,1]^d)$ with $\theta < 1$ we prove in the smoothness range $\frac{1}{p} < r < \min\{\frac{1}{\theta} - 1, 2\}$ the purely polynomial rate

$$\sigma_m(S^r_{p,\theta}B([0,1]^d), \mathbb{F})_{L_q([0,1]^d)} \lesssim m^{-r}.$$

Note all our results are obtained by constructive algorithms using coefficients that are generated by a finite number of function evaluations.

This is joint work with Tino Ullrich.

[1] Triebel, Hans . Bases in function spaces, sampling, discrepancy, numerical integration. EMS Tracts in Mathematics, 11. European Mathematical Society (EMS), Zrich, 2010. x+296 pp. ISBN: 978-3-03719-085-2